

A STANDARDIZED, MULTISCALE, FUZZY SPATIAL DATA MODEL FOR AVALANCHE TERRAIN EXPOSURE SCALE MAPPING

Eirik Sharp^{1*}, Cam Campbell¹, Grant Statham², Bryce Schroers³

¹ Alpine Solutions Avalanche Services, Squamish, BC, Canada

² Parks Canada, Banff, AB, Canada

³ Avalanche Canada, Revelstoke, BC, Canada

ABSTRACT: The Avalanche Terrain Exposure Scale (ATES) is a valuable tool for describing the severity of avalanche terrain. Traditionally developed for print media, ATES ratings are increasingly provided in digital mapping formats displayed through the web and mobile applications. While digital delivery offers many opportunities, the flexibility of digital platforms can lead users to interact with ATES data beyond their original design intent and mapping standards. Digital delivery would also benefit from a structured approach for generating and cataloging attribute data to ensure data organization, integration, and interpretation.

This paper introduces a standardized data model for ATES and a set of topological predicates for ATES-related spatial data for use with Geographic Information Systems (GIS) optimized for existing and anticipated digital use cases. In addition to data standardization, this paper proposes a more nuanced approach to ATES classification by allowing for fuzzy boundaries between features that better accommodate terrain classification uncertainty and enable more flexible mapping. An implementation of the data model is provided as an open-source PostgreSQL database using the POSTGIS extension. This implementation enables the development and storage of ATES data within a single normalized database and provides a vector representation of fuzzy membership in ATES classes. Ultimately, this approach seeks to drive more accurate and streamlined digital ATES mapping processes, optimizing avalanche risk management outcomes.

KEYWORDS: Avalanche Terrain Exposure Scale, ATES, GIS, Land Classification, Fuzzy Sets, Mapping

1. INTRODUCTION

The Avalanche Terrain Exposure Scale (ATES) (Statham et al., 2006) is a valuable tool created to describe the severity of avalanche terrain for backcountry travel. By characterizing the complexities of avalanche terrain into simplified ratings, the scale supports trip planning and route finding and can be incorporated into rule-based decision-making systems (e.g. McCammon and Hägeli, 2007). Initially intended to serve as an avalanche terrain rating for well-defined backcountry routes, an ATES zoning model (Campbell and Gould, 2014) introduced parameters and thresholds for ATES ratings to be mapped spatially. An updated version of ATES (Statham and Campbell, *in prep*) expands the system to include two new classes (i.e., non-avalanche and extreme terrain) and provides a technical model to rate avalanche terrain over various spatial representations and scales.

Early ATES data were designed to be disseminated through printed maps and, thus, were not optimized for digital distribution. However, there has been a pronounced shift towards providing ATES ratings

through digital platforms, such as web mapping applications and Global Positioning System (GPS) enabled devices in recent years. This transition to digital delivery has increased the utility of ATES data. Digitized ATES ratings can be easily maintained and updated, and the integration of ATES data with other relevant data is streamlined. These improvements allow for the automation of rule-based safety systems and enable users to superimpose ATES layers navigation tools. However, these new use cases are stretching the design of the original zoning model and mapping standards (Campbell and Gould, 2014).

ATES mapping is subject to the Modifiable Areal Unit Problem (MAUP) (Jelinski and Wu, 1996), where the results of the analyses can change based on the resolution. This flexibility can compound the uncertainty of mapped ATES features, which are often assessed at a scale of 50-250 m. While static maps account for such uncertainties by presenting data at fixed scales - typically between 1:20,000 and 1:50,000 - digital platforms empower users to set the viewing scale. Consequently, ATES maps may be consumed at greater resolutions than designed and at which they can suggest a level of precision that might not be justified.

This paper advocates adopting a probabilistic zoning model to address these scale-related challenges. Unlike deterministic methods, which assign a specific definitive classification to each area, probabilistic models can provide a spectrum of possible classifications with associated likelihoods. This approach

* Corresponding author address:

Eirik Sharp,
Alpine Solutions Avalanche Services,
Whitehorse, YT.;
tel: +1 867-335-9925;
email:esharp@avalancheservices.ca

acknowledges the inherent uncertainty in such categorizations. A probabilistic approach also allows for a more consistent representation of boundaries where landforms transition by offering a gradual shift in classification, rather than an abrupt change, that scales across different map resolutions.

Digital delivery also requires a structured approach for generating and cataloging attribute data to ensure data organization, integration, and interpretation. The current lack of a standardized data model limits the deployment of ATES. Integrating ATES datasets without uniformity can be inconsistent and inaccurate. Data redundancy can emerge, leading to unnecessary storage use and potential inconsistencies. Poor organization can also result in data errors, inefficient retrieval processes, and challenges in scalability as data volume expands. For users, a non-standardized structure can make data access difficult, and updates might inadvertently introduce inconsistencies. Finally, when collaboration is required, the absence of a unified model can impede smooth information sharing and understanding.

A standardized ATES data model offers a uniform framework for organizing and presenting ATES data, inherently fostering greater interoperability across platforms and applications. Such a model would

streamline data integration and simplify collaboration between researchers, developers, and end-users. Underpinning this data model with a rigorous set of topological rules ensures the integrity of ATES data. These rules act as safeguards, preventing anomalies and maintaining a high standard of data accuracy.

This paper presents a specific implementation of the proposed data model that supports a probabilistic zoning model to address scale-related challenges within an open-source PostgreSQL relational database. The extension to manage this platform is maintained at: https://github.com/eiriksharp-asas/ates_postgresql.

2. BACKGROUND

ATES ratings can be applied to various spatial features, including Areas, Zones, Corridors or Routes (Table 2-1). Ancillary data - including decision points, trailheads, established routes, and points of interest - often support ATES maps. ATES features have been traditionally represented as vector objects; however, with recent advances in automated assessment algorithms (e.g., Larsen et al., 2020), raster representations are becoming more common.

Table 2-1: ATES feature types and their spatial representation (Statham and Campbell, *in prep*).

ATES feature	Example application	Spatial representation
<i>Areas</i>	Rating a commonly defined region with either a well-defined geographic boundary or an ambiguous one.	Polygon or point
<i>Zones</i>	Rating a specific slope or terrain feature within a well-defined geographic boundary where ATES parameters dictate the zone boundaries.	Polygon or raster
<i>Corridors</i>	Rating a physical or conceptual path of travel between defined starting and end points with navigational freedom within a well-defined geographic boundary or an ambiguous one.	Polygon or line
<i>Routes</i>	Rating a physical or conceptual path of travel between a defined starting and end point with limited navigational freedom.	Line

Vector representations offer several distinct technical advantages over raster formats for ATES mapping. Vector data inherently represents spatial relationships, making it easier to depict interactions like linkage, proximity, and encapsulation, which is critical for understanding the distribution of ATES features over terrain. These relationships also enable more complex spatial queries such as network or proximity analyses, allowing for shortest path or lowest exposure calculations to be performed. Additionally, vector data can carry non-spatial attributes, allowing each ATES feature to store data detailing classification methodologies and technical frameworks. Furthermore, vector data can integrate easily with other datasets, such as avalanche hazard assessments, enabling rule-based terrain choice decision support systems. However, due to their

discrete nature, vector models struggle to represent uncertain or ambiguous boundaries. This is a crucial consideration in ATES mapping, where boundaries between ATES classes are not always distinct.

The increasing use of rasterized ATES data derived from automated methods (e.g., Larsen et al., 2020) signifies a significant advancement in utilizing technology for more efficient and potentially intricate ATES zoning. While raster representations complement conventional vector-based techniques, they also introduce innovative possibilities. Raster data, especially when derived from technologies like LiDAR and high-resolution photogrammetry, can provide a rich level of detail. Unlike vector data, which primarily showcases distinct classifications, raster data can depict continuous variations over space.

This capability allows raster datasets to better represent uncertainty and avoid issues like the MAUP.

2.1 Fuzzy representations of geographic boundaries

Fuzzy Set Theory (Zadeh, 1978) offers a mathematical methodology to manage ambiguous and imprecise information. Elements of fuzzy sets are allowed to belong to a set A defined on a universe of discourse X with varying degrees of membership defined by the function. $\mu_A(x), \forall x \in X$ with values between $[0,1]$. Generally, simple and computationally efficient membership functions - such as triangular functions - are used in fuzzy logic implementations, although more complex functions like Cauchy curves may also be employed.

Fuzzy Set Theory has found various applications in land classification and the spatial analysis of avalanche terrain (e.g., Ruan et al., 2002; Bühler et al., 2013; and Varol, 2022). It offers a flexible and realistic way to represent complex natural systems and a nuanced method to handle the inherent uncertainties and ambiguities of subjective landform analysis.

Cohn and Gotts (2020) proposed a model of fuzzy data appropriate to ATES mapping that describes landforms in terms of three distinct areas: 1) the core where the degree of membership $\mu(x)$ is highest (i.e., $\mu(x) \approx 1$) which can be a point, line, or polygon; 2) a transitional area surrounding the core where the degree of membership gradually diminishes (i.e., $0 < \mu(x) < 1$); and 3) outer boundary beyond which the membership drops to zero or near zero (i.e., $\mu(x) \approx 0$).

Applications of fuzzy set theory to vector data is still an emerging discipline and careful consideration must be given in choosing how to model the variable membership across the transition area to avoid surges in data volume and processing challenges. One approach is to model fuzzy regions as spatial plateaus (Schneider, 2014) represented by concentric ring buffers. Step functions can be used to assign membership values to the plateaus that characterize the fuzzy transition between a core feature and the outer boundary.

Although step functions are a computationally straightforward approach to assigning fuzzy membership across the transitional area care must be taken to balance precision and data storage requirements. Generally, fine-grained step functions necessitate more storage, while coarse step functions simplify data storage at the expense of precision. The design of the function must be carefully evaluated based on the specific needs and constraints of the application.

Topology, the arrangements between vector features in a spatial dataset, is an important consideration in

any data model. Topology predicates are established to ensure that features connect, align, and relate to each other in intended and consistent ways. These relationships are crucial in setting integrity constraints on vector data and formulating spatial queries. Since these relationships depend on an object's shape, defining topology predicates for fuzzy datasets results in additional complications since ambiguous boundaries make defining precise spatial relationships more complex. Several approaches have been advocated to extend crisp topological relations into the fuzzy context. However, since the spatial plateau model of fuzzy vectors utilizes crisp vector forms, traditional topological predicates can be adapted in most simple cases.

2.2 Data standardization

The absence of a standardized ATES data model poses several challenges. Mismatched data structures can lead to inaccuracies when merging datasets from various sources. Moreover, non-standardized approaches complicate data management due to diverse schemas and require additional processing for format conversions. This lack of uniformity also hinders easy data sharing with collaborators. Furthermore, non-standardized models might not consistently capture or store metadata, providing context about the data origins, development methods, or reliability. This omission can lead to misunderstandings or misinterpretations of the data. As technology evolves, updating non-standard data systems becomes cumbersome. A standardized model acknowledging the unique aspects of ATES data, like fuzzy boundaries, would streamline data handling.

A standardized ATES data model should adhere to several core principles:

- **Consistency:** Every ATES dataset should follow the same foundational structure regardless of the region or source.
- **Scalability:** The model should handle both small-scale and large-scale data and be flexible enough to accommodate increasing data volume or additional feature types without requiring significant overhauls.
- **Interoperability:** Adhering to common GIS data standards and protocols ensures that ATES data can be easily shared and integrated across various platforms.
- **Accuracy and Precision:** The model should prioritize accurate representation of terrain. Precision ensures that users can make informed decisions based on the data. While fuzzy set theory can account for uncertainties, the underlying data should be as precise as possible to provide a clear foundation.
- **Incorporation of Raster and Vector Data:** The model should efficiently integrate both vector

and raster data, ensuring a comprehensive representation of terrain.

- **User-centric Design:** Consideration should be given to the end-users of ATES maps. The design should ensure that the data is easily interpretable and involve intuitive categorizations to support clear legends and the integration of commonly recognized symbols such as those proposed by Engeset et al. (2022).
- **Inclusion of Metadata:** The model should support the integration of metadata - including data sources, collection methods, creation date, and any pertinent notes or annotations. This will allow users to gauge the data's reliability and relevance.

3. DATA MODEL

An implementation of an ATES database in the PostgreSQL relational database management system using the POSTGIS extension was developed to establish a standardized ATES data model. The pgSQL queries to generate the data model are packaged as a PostgreSQL extension maintained at https://github.com/eiriksharp-asas/ates_postgresql. The implementation is based primarily on vector data structures (Tables 3-1 and 3-2) optimized for terrain classification queries. Materialized views store the results of the spatial queries that define the fuzzification of ATES features and that join ATES features to the relevant technical classification schemas, ensuring fast reads. The implementation provides a wide

range of functions that empower users to run complex queries directly within the database, leveraging the data's spatial and non-spatial attributes. Data integrity and consistency are achieved through the normalization of the data model (Figure 3-1). PostgreSQL is known for its scalability, allowing the system to effectively handle increases in data volume associated with the fuzzification of ATES features, and PostGIS supports a wide variety of spatial data formats, making it easy to share data to and from other GIS software and systems.

Vector data is collected, stored, and distributed as single-part features in unprojected WGS84 - EPSG / SRID: 4326. However, PostGIS provides functionality to project the datasets as required. Raster data can also be stored and distributed in a location appropriate projected coordinate system.

ATES features are fuzzified by generating a transitional area from a parameter d representing the assessment confidence in meters stored as an attribute (*precision_m*). For polygon features, this value is used to generate a buffered region, S_{-d} , of width, e , interior to seed feature, S , representing the core plateau with a membership value of $p \approx 1$. For linear features, the core is represented by the seed feature without modification. A set (eq. 3.1) of x (typically 20) concentric ring buffer regions of width, $2x/d$, is generated around the core to represent spatial plateaus of decreasing membership probability over the transitional region.

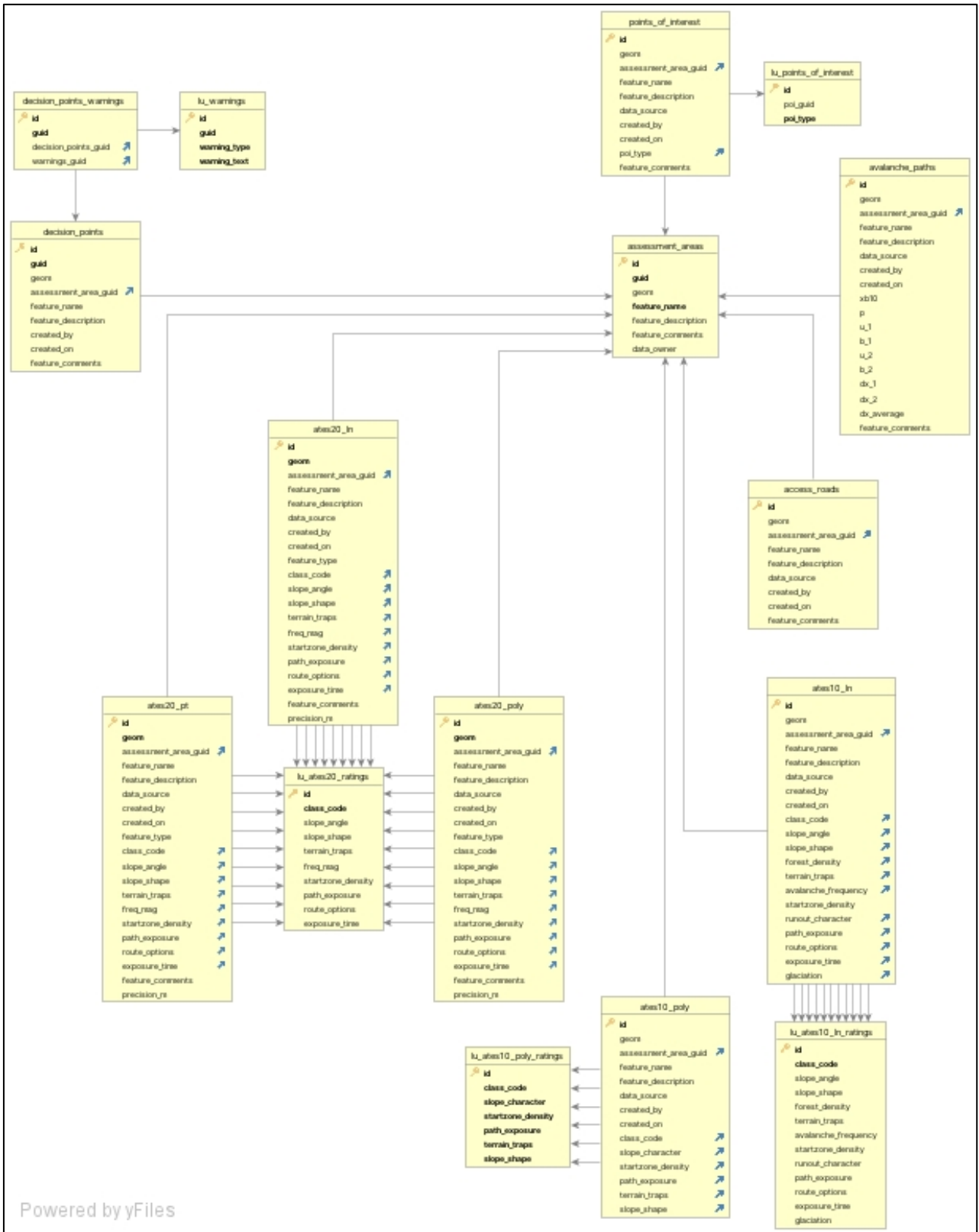
$$T = \{I_n | n \in \mathbb{N}, n \leq x\} \quad (3.1)$$

Table 3-1: Description of non-spatial data tables in the ATES data model.

Feature	Description
<i>decision_point_warnings</i>	Establishes the many-to-many relationships between <i>decision_points</i> and <i>lu_warnings</i> .
<i>Lu_ates_20_ratings</i>	Look up table detailing the ATES 2.0 classification scheme.
<i>lu_points_of_interest</i>	Lookup table cataloguing points of interest types.
<i>ly_warnings</i>	Lookup table cataloguing travel concerns and mitigations.

Table 3-2: Description of spatial data tables in the ATES data model.

Feature	Geometry	Description
<i>access_roads</i>	Line	Well-defined travel paths (e.g., road or trail) within an ATES assessment area that do not have an intrinsic ATES rating.
<i>assessment_areas</i>	Polygon	The closure of an ATES assessment area.
<i>ates20_ln</i>	Line	Linear ATES ratings of travel paths (routes or corridors) referencing the ATES 2.0 classification scheme.
<i>ates20_poly</i>	Polygon	Areal ATES ratings of (zones or areas) referencing the ATES 2.0 classification scheme.
<i>ates20_pt</i>	Point	Area ATES ratings referencing the ATES 2.0 classification scheme.
<i>avalanche_paths</i>	Line	Significant avalanche paths with easily identifiable trim lines or reasonable estimates of max runout that affect established paths of travel or have the potential to run into Class 1 terrain.
<i>decision_points</i>	Point	Locations where it is recommended to stop, regroup, and assess conditions, with recommendations on how to reduce exposure.
<i>points_of_interests</i>	Point	Locations of navigational significance, such as trailheads, campgrounds, or cabins.



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Figure 3-1: Entity diagram depicting the tables, dependencies, and normalization of the ATES data model.

Table 3-3: Description of materialized views in the ATES data model.

Feature	Description
<i>MV_ates20_areas_pt</i>	Contains the results of a one-to-many left-join of <i>ates20_pt</i> area features with <i>lu_ates_20_ratings</i> .
<i>MV_ates_areas_poly</i>	Contains the results of a one-to-many left-join of <i>ates20_poly</i> area features with <i>lu_ates_20_ratings</i> .
<i>MV_ates20_corridor</i>	Contains the results of a one-to-many left-join of <i>ates20_ln</i> corridor features with <i>lu_ates_20_ratings</i> .
<i>MZ_ates20_corridor_buffered</i>	Contains the results of a one-to-many left-join of a buffered region of width <i>precision_m</i> generated around <i>ates20_ln</i> corridor features with <i>lu_ates_20_ratings</i> .
<i>MV_ates20_corridor_fuzzy</i>	Contains the results of a one-to-many left-join over a set of concentric ring buffers of width <i>precision_m/10</i> generated around <i>ates20_ln</i> corridor features with <i>lu_ates_20_ratings</i> .
<i>MV_ates20_routes</i>	Contains the results of a one-to-many left-join of <i>ates20_ln</i> route features with <i>lu_ates_20_ratings</i> .
<i>MV_ates20_zones</i>	Contains the results of a one-to-many left-join of <i>ates20_poly</i> zone features with <i>lu_ates_20_ratings</i> .
<i>MV_ates20_zones_fuzzy</i>	Contains the results of a one-to-many left-join over the union of an interior buffered region of width <i>precision_m</i> generated around <i>ates20_poly</i> zone features with a set of 20 concentric ring buffers of width <i>precision_m/10</i> centred around this area, with <i>lu_ates_20_ratings</i> .
<i>MV_decision_point_warnings</i>	Contains the many-to-many left-join of <i>decision_point_warnings</i> with <i>decision_points</i> and <i>lu_warnings</i> compiled as a JSON array.

The union of all the buffered regions generated (eq. 3.2) is equal to S_{+d} , the buffered region of width d exterior to S , representing the area contained by boundary where the membership value drops to 0) (Schneider, 2014).

$$S_{-d} \cup T = S_{+d} \quad (3.2)$$

The membership values of these spatial plateaus are assigned according to the step function (eq. 3.3) to represent the diminishing confidence across the transition zone.

$$p_n = 1 - n/x \quad (3.3)$$

This approach provides a stepped transition between ATES zones that better represents the uncertainty of the assessment (Figure 3-2).

The topological predicates described in Table 3-3 ensure that real-world relationships are maintained within the ATES dataset and that the principles of fuzzy topology are enforced. The fuzzification of vector data and the topology of fuzzy sets remains an actively evolving domain, and a comprehensive review of all nuances and emerging methods within this field is beyond the scope of this paper. However, it is important to note that by storing the fuzzy vector data as materialized classes generated from polygons adhering to well-founded topological precepts, the architecture is anchored such that the underlying data model remains resilient and does not necessitate periodic overhauls in response to future developments in the application of fuzzy set theory to vector data.

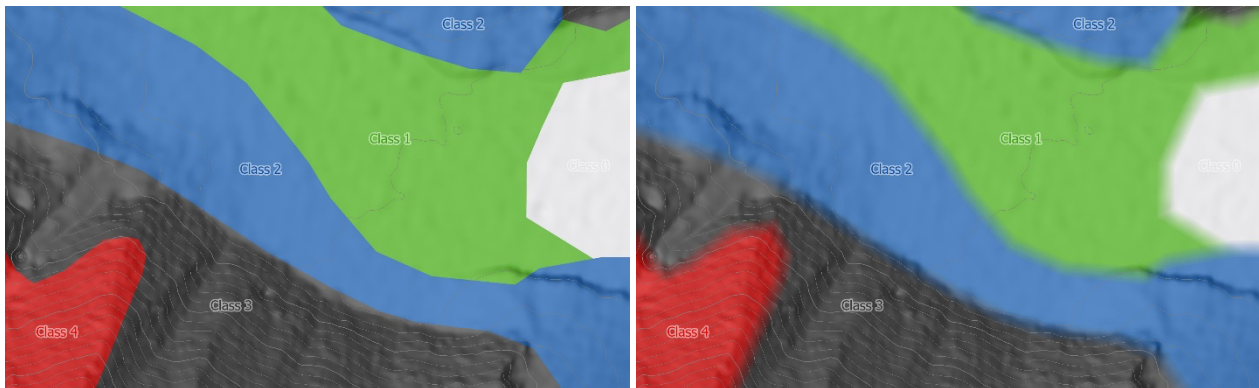


Figure 3-2: ATES zones mapped at a scale of 1:10,000 depicted with crisp boundaries (left) and fuzzy boundaries modeling an assessment uncertainty of $\pm 50\text{m}$ (right).

Table 3-3: Description of topological predicates defining the spatial relationships of features in the ATES data model.

Feature	Rule
<i>access_roads</i>	Endpoints must touch. Features must not overlap. Features must not have dangles. Features must not have pseudo-nodes. Features must intersect at a node. Features must not self-intersect.
<i>ates20_ln</i>	Endpoints must touch. Features must not overlap. Features must not have dangles. Features must not have pseudo-nodes. Features must intersect at a node. Features must not self-intersect. Features must be contained within an <i>assessment_area</i> .
<i>ates20_poly</i>	Features must not overlap. Features must not have gaps. Adjacent features must have shared boundaries. Features must be larger than a cluster tolerance defined by the uncertainty attribute. Features must be contained within an assessment area.
<i>ates20_pt</i>	Features must be contained within an <i>assessment_area</i> .
<i>avalanche_paths</i>	Endpoints must touch. Features must not have dangles. Features must not self-intersect. Features must be contained within an <i>assessment_area</i> .
<i>decision_points</i>	Features must not overlap. Features must not have gaps. Adjacent features must have shared boundaries. Features must be contained within an <i>assessment_area</i> .
<i>MV_ates20_areas_pt</i>	Features must be contained within an <i>assessment_area</i> .
<i>MV_ates20_areas_poly</i>	Features must not overlap. Features must not have gaps. Adjacent features must have shared boundaries. Features must be larger than a cluster tolerance defined by the uncertainty attribute. Features must be contained within an assessment area.
<i>MV_ates20_routes</i>	Endpoints must touch. Features must not overlap. Features must not have dangles. Features must not have pseudo-nodes. Features must intersect at a node. Features must not self-intersect. Features must be contained within an <i>assessment_area</i> .
<i>MV_ates20_corridors</i>	Endpoints must touch. Features must not overlap. Features must not have dangles. Features must not have pseudo-nodes. Features must intersect at a node. Features must not self-intersect. Features must be contained within an <i>assessment_area</i> .
<i>MV_ates20_corridor_fuzzy</i>	Features may overlap only when the fuzzy union of their membership functions is less than 1 (i.e., $\mu(A(x) \cup B(x)) = \max(\mu_A(x), \mu_B(x)) < 1$). Features must not have gaps. Adjacent features must have shared boundaries.
<i>MV_ates20_zones_fuzzy</i>	Features may overlap only when the fuzzy union (of their membership functions is less than 1 (i.e., $\mu(A(x) \cup B(x)) = \max(\mu_A(x), \mu_B(x)) < 1$). Features must not have gaps. Adjacent features must have shared boundaries.
<i>MV_decision_point_warnings</i>	Features must be contained within an <i>assessment_area</i> .

4. CONCLUSION

The increasing scale and evolving use cases of ATES data - combined with the challenges of digital storage, creation, and dissemination - demand a scalable and adaptive approach. The ATES data model proposed in this paper is tailored for such needs, enhanced with fuzzy set theory to handle the inherent uncertainties in modeling natural hazards such as avalanches. Using PostgreSQL leveraging the PostGIS extension capitalizes upon the advantages of open-source tools in managing complex spatial datasets. This choice ensures capacity handling for increasingly large data and spatial scales and anticipates future enhancements in processing fuzzy vector features.

The proposed model is modular and adaptive, designed to accommodate current requirements and future developments in GIS capabilities. The standardized structure and topological rules aim for consistent

data integrity while allowing flexibility for evolving technological and scientific shifts in avalanche mapping. Ultimately, this approach seeks to drive more accurate and streamlined digital ATES mapping processes, optimizing avalanche risk management outcomes.

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